

## Exploring Practical Work as a Sustainable Strategy in Rural Mathematics Classrooms: A Case of Addition of Fractions

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**KEYWORDS** Fraction Circles. Diagrams. Dialogue. Prior Learning

**ABSTRACT** Addition of fraction is dealt with from grade 6 onwards in South African schools topics as prescribed by the Curriculum and Assessment Policy Statement. This paper reports on a study exploring practical work as a strategy to sustain rural mathematics classrooms. The conceptual perspective for the design and analysis of this study was based on a learning ecology framework consolidated by socio-culturalism and situated learning. This study was a qualitative case study (n = 42). The participants in this study were from a rural school of the Umhlali ward, which is in the Ilembe district in KwaZulu-Natal, South Africa. The learners engaged in four activities in four lessons. They used either diagrams or fraction circles to complete the tasks in the four activities. The researcher analysed the written responses, observed their working in groups and carried out interviews. This case study found that practical work, in a rural setting, was a suitable strategy to sustain an effective mathematics learning ecology.

### INTRODUCTION

Moodley et al. (1992) and Brijlall et al. (2006) highlighted the relevance of Mathematics, insisting that the academic content of this subject becomes valuable to a learner if it provides a valuable context for thinking about and using a particular aspect of Mathematics outside the school environment or in the world of work. Also, Ernest (1998) indicated that mathematical education at school ought to make students aware of themselves in relation to other people. Mathematics should make students perceive themselves as citizens of a country with certain obligations. These obligations carry with them certain responsibilities and require an attitude of understanding and patriotism. For that reason, Turner et al. (2011) suggested that linking mathematical ideas and procedures in a context that students' value motivates and develops their critical consciousness, thus linking Mathematics ideas to the depth of understanding and range of contexts in which they apply the Mathematics lens. In this paper the researcher provided opportunity for learners to engage practically with fractions. After such explorations the learners will hopefully become prepared for enacting the more abstract cognitive traits required for mathematical problem solving. Fujita and Jones (2014) in their paper showed how emphasis on practical activity could encourage learners to form conjectures. Their study was on ge-

ometry whereas this paper reports on learners working with fractions.

Chaplain (2003) suggested that the classroom should be a pleasant environment where learners' interest is captured. To earn psychological investment (educators' expectation) educators have to work hard in producing a positive atmosphere in the classrooms. This is achieved by developing lessons that are interesting and stimulating and providing a safe environment and appropriate support of learning. The learning atmosphere changes learners in different ways: Some learners are influenced by classroom layout, seating, temperature and smell as well as the quality of learner- educator interaction in class (Chaplain 2003). Morcom (2014) in her study demonstrated how explicit values education scaffolded social and emotional learning and promoted a sense of togetherness. The researcher in this paper took the lead from Morcom (2014) and encouraged learners to work collaboratively.

Mathematics learners are expected to think creatively. Classroom environment is crucial to the fostering of creative ability. An environment full of ideas, experiences, interesting materials and resources can stimulate creativity (Craft et al. 2001). The décor and organization of a classroom should transmit what one expects to be going on in class. The mathematics educator should link theories learnt in mathematics (especially fractions) to the real world. In a mathematics class, posters and concrete objects on

display are useful. They can rouse interest from learners and assist them in making sense of mathematics. Of course besides creating the appropriate environment for effective learning, it is expected that the mathematics educators have a thorough understanding of the pedagogical content knowledge (Brijlall and Maharaj 2014).

The layout of the classroom also affects communication in the classroom. Eye contact, social distance, posture and gesture can all be enhanced by attention to the classroom layout. Some learners can easily feel excluded because of where they are positioned in class. To avoid this happening, the educator should reflect on who is sitting where and the reason. This exercise develops a positive relationship with learners who are at risk of social exclusion. Where the individuals are asked to sit, the nature of work they are given, the degree to which they are empowered to ask questions in class and emotional warmth of the environment all have an influence on learners. The above influences how students think, learn and feel about themselves and how they subsequently behave in class. Organizing the classroom directly influences both the nature of the interaction and the style of teaching and in addition should match the educators' behavioural goals. This is the type of learning ecology we kept in mind when designing this study

### **Using Practical Work to Create a Sustainable Mathematics Learning Ecology**

Practical work in fractions involves giving learners different representations and models to manipulate. It also entails hands-on activities, involving manipulation of concrete objects and drawing diagrams and pictures. Witherspoon (1993) suggested that to gain complete understanding of fractions learners needed to be exposed to the following representations: symbols, concrete models, real-life situations, pictures and spoken language.

Representations should be treated as essential elements in supporting students' understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to oneself and to others (Brijlall et al. 2011). Concrete models are critical forms of representation and are needed to support students' understanding of, and operations with, fractions. Other important repre-

sentations include pictures, contexts, students' language, and symbols. Translating among all these representations makes ideas meaningful to students. We propose that these representations are achievable in rural settings and also sustain a mathematics learning ecology.

The Curriculum Assessment Policy Statement (CAPS) (DoBE 2011) describes mathematics as the subject that assists learners to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute in decision-making. To achieve this, CAPS suggest active and critical learning amongst learners thus encouraging an active and critical approach to learning, rather than rote and uncritical learning of given truths. Learning of mathematics aims at giving learners an opportunity to gain deep conceptual understandings in order to make sense of mathematics and acquisition of specific knowledge and skills necessary for the application of mathematics to physical, social and mathematical problems. So, CAPS encourages practical work in a mathematics learning ecology.

It has been argued that concrete experiences were a basic constituent for practical activities. Also noting the importance of concrete experiences is Ott et al. (1991: 7) who pointed out that "familiar concrete experience, actual or recalled should be the first step in the development of new abstract concepts and their symbolisation". Dienes (1964) championed the use of collaborative, group work and concrete materials, as well as goals such as democratic access to the process of mathematical thinking, long before the dictates of constructivism. Dienes invented blocks which allowed students to explore the numeration system and how the operations on numbers were addressed by the system. Ott et al. (1991) pointed out that "...familiar concrete experience-actual or recalled-should be the first step in development of new abstract concepts and their symbolization"

According to Piaget (1985), the concrete operational stage typically develops between the ages of 7-11 years. Intellectual development in this stage is demonstrated through the use of logical and systematic manipulation of symbols, which are related to concrete objects. Thinking becomes less egocentric with increased awareness of external events, and involves concrete references. With reference to Piaget's suggestions the researcher saw practical activities as

relevant for learners in grade six. Learners in this grade are about eleven years old, meaning that they are at concrete operational stage. They need to be given an opportunity to manipulate objects in order to learn. Hence, for this study we formulated the research question:

1. *How can sustainable rural mathematics learning ecologies be created?* With this in mind we created learning ecologies involving practical work. We then asked the following sub-questions to unpack the critical research question:

2. *What are the learners' attitudes towards practical work?* This question helped to determine whether learners found it interesting to manipulate concrete objects they have been given, and whether this helped them to understand the addition of fractions better.

3. *Which materials do learners prefer between diagrams and fraction circles?* This question helped the researcher to determine which used material was most preferred by learners. This would give the researcher the opportunity to look at why the material was not preferred by learners and if justifiable to omit in the teaching process. Figure 1 illustrates one group of learners using fraction circles to carry out their tasks.

### Conceptual Framework

Barron (2006) pointed out that the learning ecology framework draws on ecological perspectives. In our case we studied the creation of a classroom environment within which the central stakeholder (the mathematics learner) interacted with the designed activities and peer learners to promote the learning of mathematics. This ecology also relied on constructs developed from sociocultural and situated learning. Such a learning ecology was organised and managed by the researcher. The researcher was thus expected to provide experiences, guide discussions and assume a supportive role in the process of learners' attempts of developing understanding.

Borrowing from (Vygotsky 1978; Breen 1992) children learn from people around them and their surroundings. Vygotsky further claimed that central to the process of learning is the role played by society in the development of higher cognitive situations. She also contends that learners work with their peers in collaborative situations or they complete tasks and/or activities with the educator as a facilitator (Vygotsky 1978). Also relevant to Vygotsky's views, as ar-



Fig. 1. Learners actively engaged in activity 1

gued by Billet and Lave (1993) cited by Mokapi (2002), that knowledge that individuals construct, organise and store in memory is embedded in the context or “community of practice” (Lave et al. 1991) where it is learned. According to Lave (1996: 64) “learning is recognised as a social phenomenon constituted in the experienced, lived-in world.” In this study, practical problem-solving skills involving fractions were explored as a strategy to sustain a mathematics learning ecology drawing from the relevant information provided by the above-mentioned theorists.

### METHODOLOGY

The researcher was granted permission to do this qualitative study at a rural school of the Umhlali ward, which is in the Ilembe district. The enrolment of the school is currently about seven hundred and sixty and there are twenty-two educators. This class consisted of forty-two mixed ability learners. The age of these learners ranged from eleven years to fifteen years. Most of them started school in the same school and they live in the farm around the school. The purpose of taking a small group of learners was to

uncover in-depth information about what happens when learners learn addition of fractions using practical means. It is argued that “Qualitative inquiry typically focuses on relatively small samples, selected *purposefully* to permit inquiry into and understanding of phenomenon in *depth*” (Cohen et al. 2007)

During teaching learners were seated in groups (see Fig. 2). This arrangement allowed them to work together and help one another to solve problems. Problems were written on activity sheets. This gave the researcher an opportunity to move around and observe learners interacting with one another. After group work, learners were given enough time to complete their individual activity sheets. Each activity sheet consisted of ten problems to solve. While learners were working individually the researcher observed learners and helped those learners who were struggling. All assessed scripts were collected for data capturing.

### Creating Mathematics Learning Ecologies

With the assumption that these learners had been exposed to fractions before, although not addition of fractions with different denomina-



Fig. 2. Learners in action

tors, the decision was to engage them in four lessons on four consecutive days. On each day the lesson involved the learners engaging with different activity sheets.

Activity one was on “*addition of fractions with the same denominators*”. In this activity learners were asked to use fraction circles to find answers. This was done with an intention that learners should find it easy to add pieces that are the same, as this would capture their interest. Also, another aim was to prepare them for the next activity, in which the researcher wanted them to discover for themselves that: 1) it is not possible to add fractions with different denominators, 2) to add fractions it is always important to make denominators to be the same, 3) once the denominators are the same only the numerators are added not the denominators, 4) if the numerator is the same as the denominator, that makes a whole and 5) answers are always left in their simplest form, as this would help them to do factorisation in the senior phase.

Activity two dealt with “*addition of fractions with different denominators*” (one denominator is a multiple of the other). Each group was given its own problem. The researcher gave them fraction circles to solve problems. Learners tried to match pieces that they were adding, but they were unable to come out with the final answer. They did not know whether the answer was to be in sixths or thirds. It was very interesting to note that some groups were able to see that one third equals to two sixths. It was at that time that diagrams were introduced. Diagrams helped learners not only to learn addition of fractions, but also: 1) the concept of equivalency, 2) factors and multiples of numbers, 3) drawing correct diagrams to represent fractions and 4) simplification of fraction

The third activity involved “*addition of fractions with different denominators*” (none of the denominators is a multiple of the other). In this instance diagrams were used to: 1) consolidate equivalency and 2) to demonstrate that different numbers can have the same multiples.

For activity four the tasks were on “*addition of mixed fractions*”. In this case they discovered: 1) that a whole is made of eight pieces if we talk about eighths using fraction circles and 2) how improper fractions are formed.

The research instruments that were used to bring out the required data were specifically associated with qualitative studies. These instru-

ments were: 1) semi-structured observation during teaching, 2) semi-structured interviews with learners and 3) questionnaire.

## RESULTS AND DISCUSSION

From observing learners’ marks, it was evident in that in the class of forty-four learners all learners got correct answers for activity one in lesson one. The learners used fraction circles, as they are colourful and easy to handle. They made it easy for the researcher to explain that if you add one-eighth and two-eighths for example they add up to three-eighths. This eradicated the learners’ misconceptions (in majority of the cases) that if you add fractions you also add denominators. In Table 1 the researcher shows a record of learners’ marks for activity one. The researcher notes that for this activity no learner got any task wrong. The majority (over ninety two percent) of the learners got the tasks completely correct. For this task learners used fraction circles and these seemed to have contributed to the good performance of the learners.

**Table 1: Performance in activity one**

Question number	Corret response	Partially correct response	Incorrect response
1	39	3	0
2	40	2	0
3	42	0	0
4	42	0	0
5	41	1	0
6	42	0	0
7	41	1	0
8	42	0	0
9	41	1	0
10	40	2	0

The researcher shows in Figure 3 Learner L28 who was coded as “partially correct”. For this task L28 added both the numerators and denominators of the two fractions to get an answer of  $\frac{6}{16}$  (of course a further error in simplification resulted in  $\frac{2}{8}$ ). Learner L28 added the denominators in task 1 as well. However, she

$$\frac{3}{8} + \frac{3}{8} = \frac{6}{16} = \frac{2}{8} = \frac{1}{4}$$

**Fig. 3. The written response of learner L28 to task 9 of activity one**

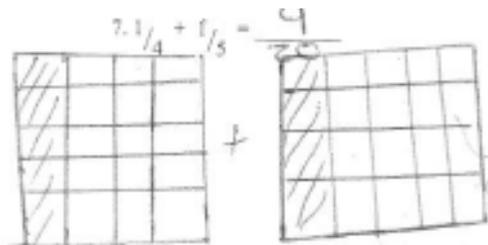
got the other eight tasks correct. This could mean that she was reasonably successful in employing concrete objects (in this case fraction circles) to add fractions in activity one.

For Activity two more than ninety percent (see Table 2) of the learners got each task correct. This activity expected the learners to engage in using diagrams as a practical tool to solve the tasks. To highlight this category of responses we explore the written response of learner S10 in Figure 4.

**Table 2: Performance in activity two**

Question number	Corret response	Partially correct response	Incorrect response
1	42	0	0
2	42	0	0
3	38	0	4
4	42	0	0
5	42	0	0
6	40	0	2
7	42	0	0
8	38	0	4
9	42	0	0
10	39	0	3

Learner L10 firstly decided to use two large rectangles and secondly divided them into twenty squares. The researcher presume that he counted all the shaded squares and attained nine such and hence realised the number of shaded squares as a fraction of the total number of squares was  $9/20$ . We note that the diagrams aided conceptual understanding of the operation of fractions in this task. In fact those teachers who prefer algorithms could use diagrams as a basis to introduce the use of the lowest common denominator (LCD), in this instant using 20 as an LCD of 4 and 5. In using diagrams one should bring to the attention of the learners that the initial rectangles used must be of the same



**Fig. 4. The written response of learner L10 to task 8 of activity two**

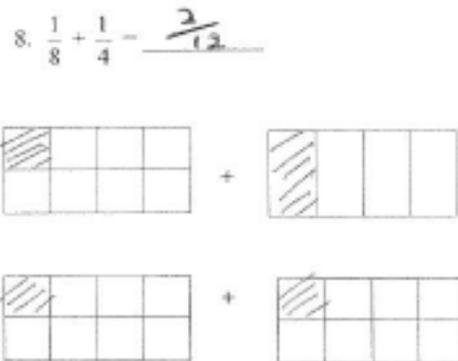
size and that so should the smaller squares. For teachers to design tasks to address a variety of suitable mathematical competence and skills it is expected that these teachers have a firm grounding in their pedagogical content knowledge (Brijlall and Maharaj 2014).

For activity three all the learners got tasks 1 and 2 correct (see Table 3). Task 7 was relatively poorly answered. This task was: Add  $1/4 + 1/5$ . We analyse the written response of learner L1.

**Table 3: Performance in activity three**

Question number	Corret response	Partially correct response	Incorrect response
1	42	0	0
2	42	0	0
3	41	0	1
4	41	0	1
5	38	0	4
6	31	10	1
7	38	0	4
8	39	0	3
9	39	3	0
10	37	0	5

Learner L1 managed to use the first pair of rectangles to denote the fractions  $1/8$  and  $1/4$  correctly. It seemed that L1 did not go back and reflect on the shaded squares in the second pair of rectangles. She therefore counted the two shaded squares. It also seemed that she merely added the denominators 8 and 4 to get 12.



**Fig. 5. The written response of learner L1 to task 7 of activity three**

Activity four was designed with tasks that involved the addition of mixed fractions. For this activity learners engaged the tasks using either practical tool (fraction circles or diagrams). Task

1 provided no difficulty to the learners (see Table 4). However, the overall performance in this activity was the worst when compared with the previous three activities. The worst performed task was task ten. We show the written response of learner L38 to task 10 in Figure 5.

**Table 4: Performance in activity four**

Question number	Corret response	Partially correct response	Incorrect response
1	42	0	0
2	40	0	2
3	30	10	2
4	28	11	3
5	31	12	1
6	37	5	0
7	36	5	1
8	35	1	6
9	37	0	5
10	28	8	6

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In Figure 6, learner L28 did not represent the mixed fractions by correct diagrams. For instance

the mixed fraction  $2\frac{1}{2}$  should have been represented as



Instead she drew two rectangles and then inserted the half of a rectangle into the one already drawn. This could mean that she did not have a complete conceptual understanding of the concept of a mixed fraction from her previous years learning. So, present learning needs to take into cognisance prior learning to contribute to an effective mathematics learning ecology.

**Learner Perception on Practical Work**

To gather information about the learners' perceptions to practical work, the researcher observed the behaviour of the learners. Their facial expressions were observed, for example as young learners when they encounter problems one could read from their faces. Some do frown or smile when they were pleased and enjoyed the task they were given. In this class everybody even those who were shy communicated with others and showed via their facial expressions that they enjoyed what they were using, particularly the fraction circles. Enjoyment in classroom activity is shown to foster appreciation for learning (Morcom 2014). Some learners did struggle a bit with diagrams, but finally got into understanding how to draw them. In order to gather the learners' perception or attitudes towards practical work we carried out interviews with learners. We denote the researcher by I and the learner by L. The following dialogue ensued with learner L1 and L3:

I: What did you understand about doing practical work when learning fractions?

10.  $1 + 2\frac{1}{2} + 3\frac{1}{2} = 4\frac{1}{2}$



**Fig. 6. The written response of learner L28 to task 10 of activity four**

L1: In practical work we used fraction circles and drew diagrams to find answers to the questions.

I: What did you understand about doing practical work when learning fractions?

L3: It was when madam you asked us to use the pieces of fractions to complete our activities. We sorted these pieces to find answers.

These learners showed that they understood what entailed practical work. To them it was about using fraction circles and diagrams to find answers in addition of fractions. To enquire whether learners previously engaged with practical work, learners L1 and L4 provided the following verbal responses to queries raised by the researcher:

I: Have you been engaged in practical when you learn addition of fractions? If yes, what did you use and how did you use that?

L1: No. Our teacher in grade 5 taught us to find the denominator of the two numbers.

I: Have you been engaged in doing practical when you learn addition of fractions? If yes, what did you use and how did you use that?

L4: No. I did not know practical work before.

This typical response highlighted the rare use of practical work by teachers.

Responses to the preferences of fraction circles to diagrams indicated that most learners preferred using fraction circles as compared to diagrams. This we infer from a dialogue like:

I: In the four lessons that we had, we used fraction circles and diagrams. Which one did you find interesting to use? Explain for each one.

L3: I loved both of them, but I liked fraction circles more.

I: In the four lessons that we had, we used fraction circles and diagrams. Which one did you find interesting to use? Explain for each one.

L4: I liked using fraction circles

I: Why?

L4: They were easy to use and they are beautiful

For the last question learners' responses showed that they enjoyed using practical work. They said everything became very easy for them. The following are some responses which demonstrated this:

I: (Patting the learner's shoulders) You tried very hard, okay let's look at each ques-

tion. In question 5 your diagrams are correct, but when you wrote your final answer, you wrote your denominator as 7 instead of 14, as you diagrams suggest. Why?

L4: Hawu! ngenzeiphutha. I made a mistake

I: Also your question 7 your first diagrams are correct, with second diagrams one is correct, but the other one has thirty-eight pieces instead of twenty spaces. How come.

L4: I don't want to lie, madam I was exhausted, I was lazy to count.

I: Oh! no your laziness has costed you marks.

L4: I m happy madam, because I know these sums.

I: Okay, please we still have activity 4 to do make sure you complete it.

L4: Thanks

## CONCLUSION

Starting from what learners know played a crucial role in introducing the series of lessons that learners were going to be engaged in. In grade six actually they were supposed to add fractions that have different denominators where one is the multiple of the other. But our reasons for starting with addition of fractions with similar denominators were to identify misconceptions that they had in addition of fractions and to capture their interest in the lessons that they were to be engaged in the next coming days.

This paper also found that the practical work had promoted successful addition of fractions.

There are reasons why concrete objects are also recommended in the teaching of addition of fractions. One of the reasons is that learners' learning styles are not the same. Some are auditory, some kinetic, some are visual and some learn better if they do. So concrete objects facilitate understanding to those types of learners. In order to keep learners' attention in the lesson sometimes learners have to be engaged in hands on activities. For young learners materials should be colourful and durable for their use. At college the researcher learnt that when learners do activities they remember them, it is unlike when they see them or hear them.

The analysis of the data for this study also found that dialogue within the mathematics learning ecology led to better conceptual understand-

ing of the addition of fraction. This was evident to most of the learners worked collaboratively, when they drew their diagrams they just drew without taking note of the fact that fractional pieces should be equal and also labelling fractions did not matter to them. It was noted only when they were with the researcher assessing at each script, that learners realized the importance of accurate drawing and writing of fractions. The researcher observed that some learners got lost when the researcher was using English throughout.

Although this paper reports on a small scale case study the researcher admits that the findings cannot be generalised. This case study found that practical work, in a rural setting, was a suitable strategy to sustain an effective mathematics learning ecology.

### RECOMMENDATIONS

The researcher provided tasks which started with addition of fractions with similar denominators in order to identify misconceptions that the learners had in addition of fractions and to capture their interest in the lessons that they were to be engaged in the next coming days. The researcher therefore recommends that learning of previous concepts be considered before proceeding with extensions of these concepts to create effective mathematics learning ecologies.

The analysis of the data for this study also found that dialogue within the mathematics learning ecology led to better conceptual understanding of the addition of fraction. The researcher recommends that dialogue between the teachers and learners should be done throughout the lesson. This develops the trust between the two participants and most importantly it gives the teacher the opportunity to investigate learners' conceptual understanding of mathematical concepts.

The researcher also recommends that learners be allowed to speak in languages they were comfortable with especially in multilingual classes. The researcher observed that the learners began to be relaxed when code switching was used. Code switching is recommended, because some terms are missing and difficult in vernacular and also some are explained clearly in vernacular. Academics have argued that there are two types of knowledge acquisition: spontaneous and scientific or scholastic. Teachers have

to see to it that they identify learners' knowledge about each concept.

### ACKNOWLEDGEMENTS

The researcher would like to thank Ms Fortunate Gugulethu Mdluli for assisting in the data capture. The researcher would also like to place on record his sincere appreciation for the proof-reading of this paper by his colleague Mrs E Dowlath.

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